EFFECT OF A SURFACTANT ON THE DYNAMICS OF OIL DROPLET NEAR A SOLID SUBSTRATE

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Motivation

- The presents of surfactants in the water injection in Enhanced Oil Recovery (EOR)can increase the oil production [Xie (2004), Wu et al. (2008), Krisanto et al. (2010)].
- Surfactant can decrease the interfacial tension between water and oil [Oron, et.al. (1997), Myers (1998), Leal (2007)].
- When the oil adheres to the solid surface, surfactants can alternate the wettability of surface [Abdallah (2007), Kristanto, et.al (2010), Zhang, et.al (2006)].

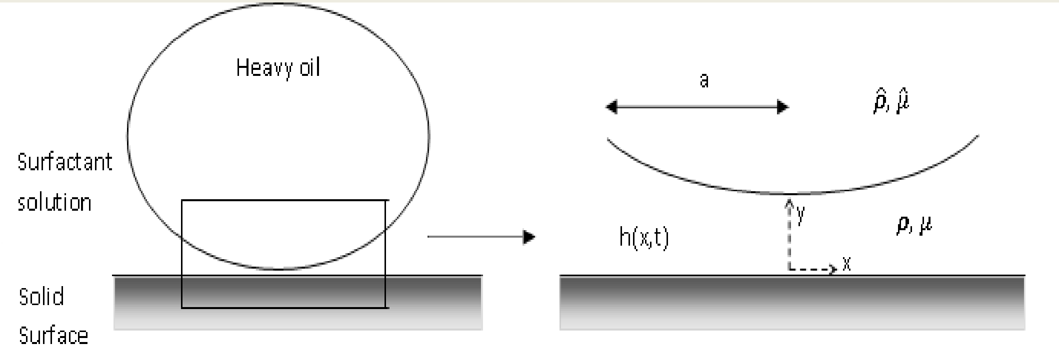
Questions and goals

Q: When oil has detached the rocks surface, will it get back to the rocks surface or remain in its initial position.

G: to construct a mathematical model that describes the influence of surfactants on the dynamics of the thin film which is formed by an oil droplet (viscous liquid) and a solid surface. More specifically:

- will the oil droplet near a solid surface immersed in the surfactant solution adhere to the solid surface in finite time?
- What factors that affect the stability of the oil droplet in the surfactant solution?
- What is the optimum surfactant concentration such that the oil droplet moving away from a solid surface?

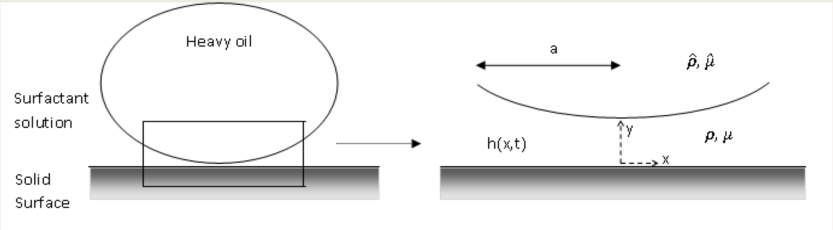
Geometry



The thin film between a solid surface and a drop

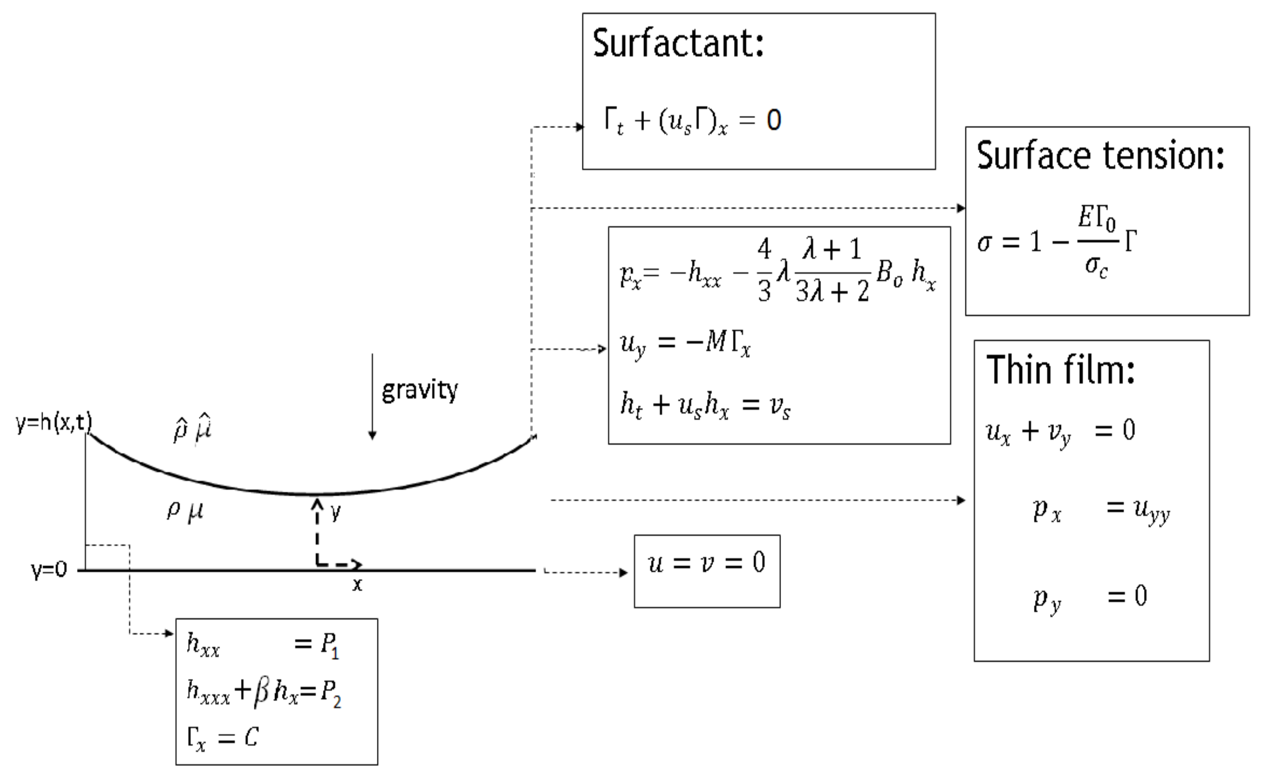
x	The horizontal coordinates	y	The vertical coordinates
u	The horizontal velocity	v	The vertical velocity
μ	The thin film's viscosity	ρ	The thin film's density
h	The thin film's height	t	The time

Assumptions



1. The thin film and the drop are incompressible viscous Newtonian fluids, such that the 2-D creeping flow approximation is eligible.
2. The motion of the drop is perpendicular to the horizontal smooth solid surface.
3. The drop is symmetry to the normal axis.
4. Surfactant is insoluble and distributed at interface of thin film-liquid drop by convection.
5. Surfactant concentration is sufficiently small and affects only on the drop surface, without any more complex dynamical or rheological effects.

The governing equations (dimensionless form)



Surfactant:

$$\Gamma_t + (u_s\Gamma)_x = 0$$

Surface tension:

$$\sigma = 1 - \frac{E\Gamma_0}{\sigma_c}\Gamma$$

$$p_x = -h_{xx} - \frac{4}{3}\lambda\frac{\lambda+1}{3\lambda+2}B_o\,h_x$$

$$u_y = -M\Gamma_x$$

$$h_t + u_s h_x = v_s$$

Thin film:

$$u_x + v_y = 0$$

$$p_x = u_{yy}$$

$$p_y = 0$$

$$u = v = 0$$

$$h_{xx} = P_1$$

$$h_{xxx} + \beta h_x = P_2$$

$$\Gamma_x = C$$

Reduced equations

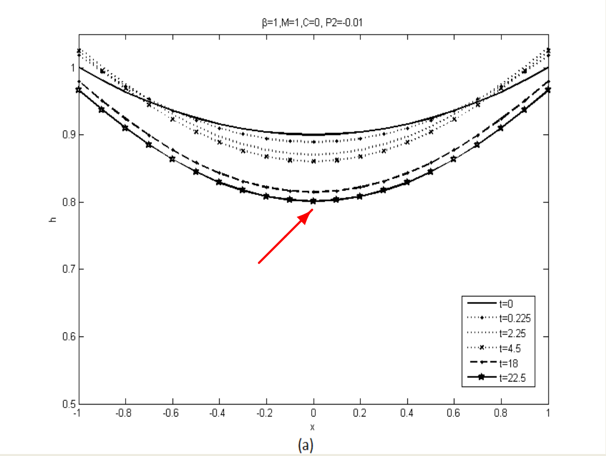
$$
\begin{aligned}
h_t &= [\frac{1}{2}M\Gamma_x h^2]_x + [\frac{1}{3}h^3(h_{xxx}+\beta h_x)]_x, \qquad (1)\\
\Gamma_t &= [Mh\Gamma\Gamma_x]_x + [\frac{1}{2}\Gamma h^2(h_{xxx}+\beta h_x)]_x. \qquad (2)
\end{aligned}
$$

where $\lambda = \frac{\widehat{\mu}}{\mu}$, the $\beta = \frac{4}{3}\frac{\lambda^2+\lambda}{3\lambda+2}\frac{(\widehat{\rho}-\rho)ga^2}{\sigma_c}$, and the Marangoni number $M = \frac{E\Gamma_0}{HP}$.

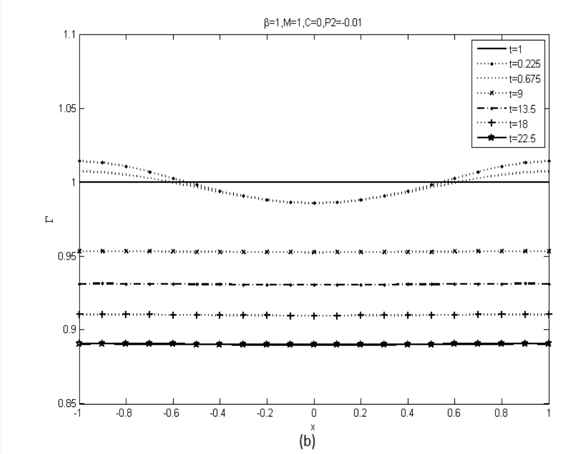
Given boundaries and initial conditions:

$$
\begin{aligned}
h(x,0) &= K + \frac{P_1}{2}x^2. \qquad (3)\\
\Gamma(x,0) &= 1, \qquad (4)\\
h_{xx}(\pm 1,t) &= P_1, \qquad (5)\\
(h_{xxx}+\beta h_x)(\pm 1,t) &= \pm P_2, \qquad (6)\\
\Gamma_x(\pm 1,t) &= \pm C. \qquad (7)
\end{aligned}
$$

Simulations:no adding rates



The height of the thin film

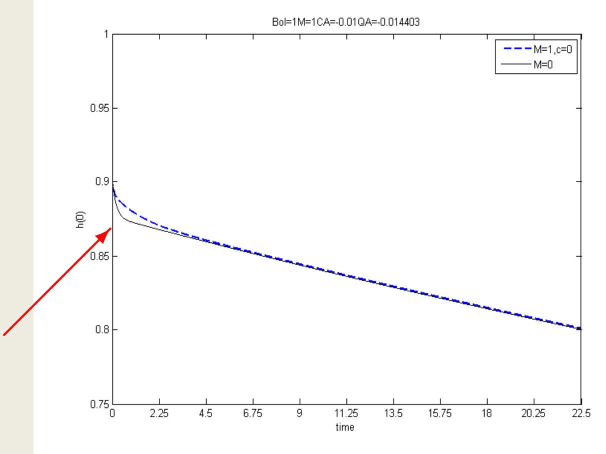


The surfactant concentration

The dynamics of fluids without surfactant adding rates, for $\beta = 1$.

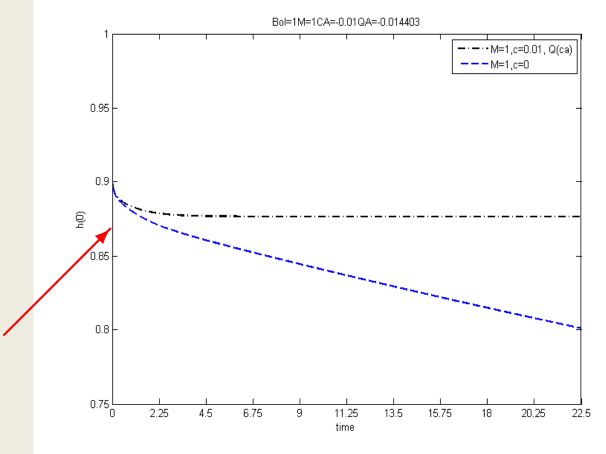
Next, we shall focus on the dynamics of the lowest part of droplet ($h(0,t)$) (see the red arrow)

Free-/covered-surfactant without adding rates



The dynamics of the thin film at $h(0)$ for free-surfactant (solid line) and covered-surfactant (dashed line), for $\beta = 1$ at dimensionless time interval [0, 22.5].

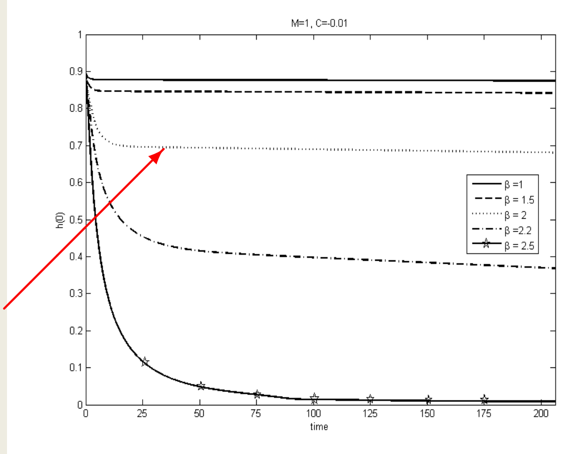
Surfactant without and with adding rates



The dynamic of thin film height without adding rate $C = 0$ (blue) and with adding rate $C = 0.01$ (Black).

For case $\beta = 1$, the effect of the adding rate to the dynamic of the thin film height is observable.

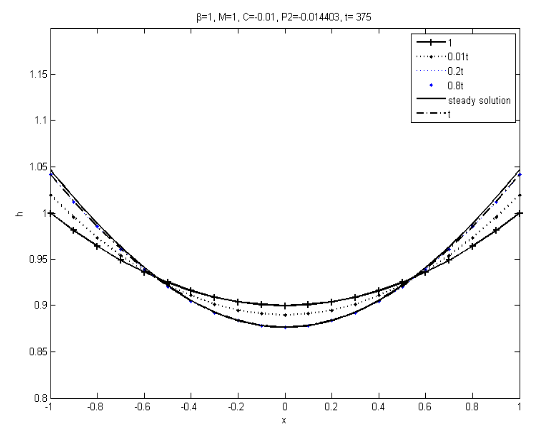
Effect of β-parameter



The dynamic of thin film height for constant $C = 0.01$ and varies values of β.

There is a limitation value of β for which an adding rate works effectively to lift the liquid drop.

Steady state case



The agreement between the numerical simulation and the steady analytical solution for height of the thin film ($\beta = 1$)

Summary

- We constructed a mathematical model, explored numerical simulations, and derived steady state analytical solution (by means of asymptotic expansion), for the influence of the insoluble surfactant on the dynamics of thin film located between a solid surface and a liquid drop.
- The present of the surfactant showed to delay the decrease of the film thickness, but only at early time.
- However, when an amount of the surfactant was added into the system, the film thickness increased significantly (in the limitation of the value of β).
- For steady state case, the numerical and analytical asymptotic solutions were both in agreement.

Output

The Dynamics of an Insoluble Surfactant-Covered Thin Film Confined between a Moving Liquid Drop and a Solid Surface

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Received: date / Accepted: date

Abstract The evolution of an insoluble surfactant-covered thin liquid film flowing between a solid surface and a liquid drop is investigated. The gravity, the viscosity ratio and the density difference of fluids, and the surfactant concentration are included. Using the lubrication approximation, the model is reduced to a set of nonlinear partial differential equations. Since we are interested in the role of the surfactant to lift up the drop, along the paper we assume that the density of the drop is higher than the density of the thin film. The equations are solved numerically by the finite-difference method. Results show that the present of the surfactant tends to delay the decrease of the film thickness, but only at early time; Later on, it behaves as same as the free-surfactant system. However, when an amount of the surfactant is added into the system, it tends to increase significantly the film thickness. For steady state case, an analytical solution is derived by using the asymptotic expansion method. For this case, our numerical and analytical solutions are both in agreement.

Keywords Thin film · liquid drop · surfactant · steady state

Mathematics Subject Classification (2000) 76D08 · 35K52 · 74G10 · 76M20

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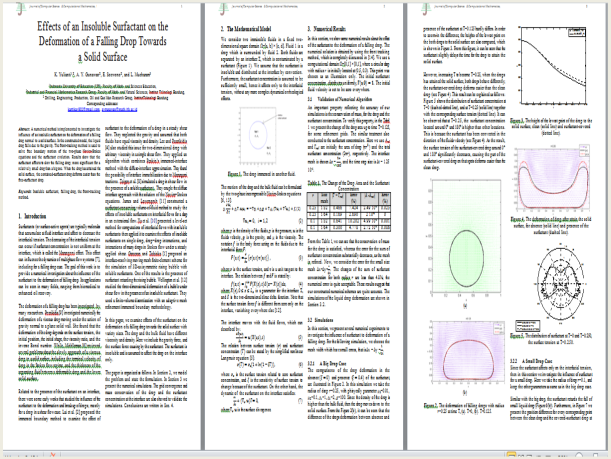
Submitted to the J. of Engineering Math. (2 monts ago)



Effects of an Insoluble Surfactant on the Deformation of a Falling Drop Towards a Solid Surface

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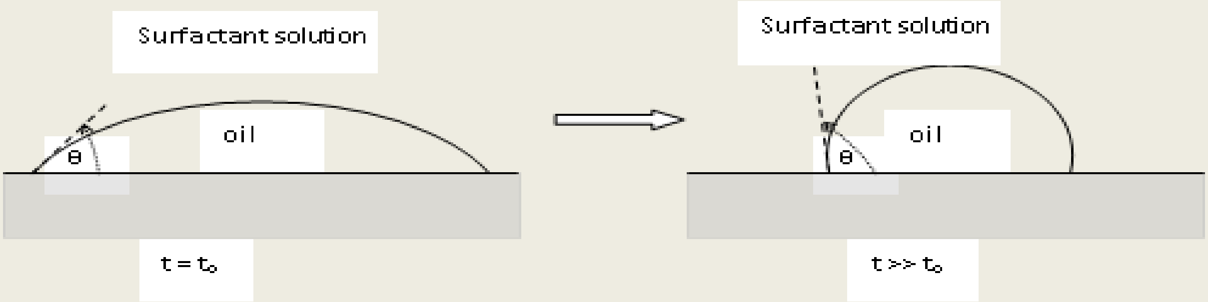
Submitted to J. of Computer Science and Computational Mathematics (last year)

Ongoing research



A wetting angle model for an oil drop on a solid surface immersed in surfactant solution, supported by The Asahi Glass Foundation 2014.

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